Modal Analysis of Flexible Aircraft Dynamics with Handling Qualities Implications

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A multivariable modal analysis technique is presented for evaluating flexible aircraft dynamics, focusing on meaningful vehicle responses to pilot inputs and atmospheric turbulence. Although modal analysis is the tool, vehicle time response is emphasized, and the analysis is performed on the linear, time-domain vehicle model. In evaluating previously obtained experimental pitch tracking data for a family of vehicle dynamic models, it is shown that flexible aeroelastic effects can significantly affect pitch attitude tracking performance. Consideration of the eigenvalues alone of both rigid-body and aeroelastic modes does not explain the simulation results. Modal analysis revealed, however, that although the lowest aeroelastic mode frequency was still three times greater than the short-period frequency, the rigid-body attitude response was dominated by this aeroelastic mode. This dominance was defined in terms of the relative magnitudes of the modal residues in selected vehicle responses.

Introduction

HISTORICALLY, the subjective opinions from highly trained test pilots on the performance and workload involved in performing missions were correlated with model-based parameters such as damping and frequency of the vehicle's rigid-body modes. These parameters were then used to infer the handling qualities of future vehicles, or for handling qualities specifications.[1]

The hypothesis taken here is that handling qualities are a property of the vehicle's dynamic behavior, or time response. The significance of a mode's frequency or damping, for example, lies in its effect on the time response. This simple fact is consistent with the classic approach being successful for rigid, unaugmented aircraft, while high-order dynamic effects (aeroelastic or control system dynamics) have been found to so significantly affect the pilot opinion as to render the approach useless.

The purpose of the work presented in this paper is to address the question, "Can dynamic aeroelastic effects significantly affect aircraft handling qualities, and if so, how?" This question is fundamental. For example, to evaluate a vehicle through simulation, the appropriate vehicle mathematical model must be selected, so the issue of how significant the aeroelastic modes are expected to be must be addressed at the outset.

To move toward answering this question, an analytical approach for evaluating a subject vehicle's dynamics will be discussed, and the method applied to a spectrum of generic vehicles for which experimental data has been obtained.

Modal Analysis Concepts

Clearly, for systems for which a simple, rigid-body transfer function[2] is a poor model, the parameters in that transfer function cannot indicate good or bad handling qualities, for they alone cannot determine the system's time response. This is the first point with regard to the basic question posed at the outset—parameters in a vehicle dynamic model are important if they affect the system's time response significantly.

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Taking the important pitch attitude response to an elevator impulse for example, (assuming distinct eigenvalues)

$$\dot{\theta}(t) = \sum_{i=1}^{n} R_i e^{p_i t} \tag{1}$$

where n = order of the system, R_i = ith residue, and p_i = ith system pole or eigenvalue.

We have (if all modes are complex)

$$\dot{\theta}(t) = \sum_{i=1}^{N} 2|R_i| e^{-\sigma_i t} \cos(\omega_i t + \psi_i)$$

where N = number of system modes, $p_i, p_i^* = -\sigma_i \pm j\omega_i$ complex conjugate pair of poles, and $R_i = |R_i| e^{j\psi_i}$ residue for p_i. We note that the eigenvalues are fundamental in their effect on time response. The "numerator dynamics" or the interaction between the poles and zeros are also significant in terms of their effect on the residues. The zeros' locations in themselves are not important; how their position affects the residues is.

This leads to a second important point. The effects of higher-order modes may be quantified in terms of their effects on the dominant (usually rigid body) eigenvalues and residues, as well as the time response of the higher modes themselves.

The above well known facts are fundamental to the approach to be presented. For more realistic dynamics (i.e., higher-order), numerical analysis using multi-variable systems techniques is appropriate. We assume a linear state-variable representation for the vehicle is available, and although the multi-input, multi-output case (e.g., θ and n_z) will be treated, consider for now the case above, or $\dot{\theta}/\delta_E$. We have

$$\dot{\bar{x}} = A\bar{x} + B\delta_E$$

$$\dot{\theta}(t) = C\bar{x}$$

Now diagonalizing the system via the modal matrix T consisting of the system eigenvectors $\bar{v}_i$, or

$$T = [\bar{v}_1, \bar{v}_2 \cdots \bar{v}_n]$$

one obtains

$$\dot{\bar{q}} = \Lambda\bar{q} + T^{-1}B\delta_E \tag{2}$$

the vehicle dynamics expressed in modal coordinates.[3] With the pitch rate, the response of interest here, expressed in terms of the system modes as

$$\dot{\theta}(t) = CT\bar{q} \tag{3}$$

where

$$\Lambda = \text{diagonal matrix of eigenvalues, } \lambda_i = -\sigma_i \pm j\omega_i$$

$$T^{-1}B = \text{Modal controllability matrix, } \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$CT = \text{Modal observability matrix, } [\, c_1 \cdots c_n \,]$$

And the partial fraction expansion of the system's impulse response is simply

$$\dot{\theta}(s) = \sum_{i=1}^{n} \frac{c_i b_i}{s - \lambda_i}$$

obtained directly from Laplace transforming Eqs. (2) and (3), and setting $\delta(s) = 1$. We now note the residue associated with λ_i is $R_i = c_i b_i$.

Another system response of interest may be obtained directly from the residues and eigenvalues found as above. The system's step response may also be evaluated, but it is interpreted as the time integral of the response of the system to an impulse. For a linear system, these responses are identical.

Referring to Eq. (1), the integral of the pitch-rate impulse response is seen to be

$$\int_0^t \dot{\theta}(t)\,\mathrm{d}t = \sum_{i=1}^{n} (R_i/p_i)\, e^{p_i t} - \sum_{i=1}^{n} (R_i/p_i)$$

or the steady-state plus transient response, with the residues for these expressed in terms of the impulse residues R_i and the poles p_i. Therefore, we may interpret the term

$$R_i/p_i(e^{p_i t} - 1)$$

for real p_i, or the sum of two complex conjugates like the above for complex p_i, as the area under mode i's contribution to the system's impulse response. The magnitude of R_i/p_i therefore reflects not only the significance of the mode's impulse response at $t = 0$, (or R_i) but also the rate of decay of this response. The slower it decays, the larger its time integral. It is interesting to note, incidently, that $|R_i/p_i|$ depends on ω_{n_i} and not on modal damping ζ.

These terms may be derived easily from Eqs. (2) and (3) as well, since

$$\int_0^t \dot{\theta}(t)\,\mathrm{d}t = CT \int_0^t \bar{q}(t)\,\mathrm{d}t$$

For zero initial conditions and an impulsive input, the modal response is

$$\bar{q}|_\delta(t) = e^{\Lambda t} T^{-1} B$$

so

$$\int_0^t \dot{\theta}|_\delta(t)\,\mathrm{d}t = CT\Lambda^{-1}(e^{\Lambda t} - I)T^{-1}B$$

Now, since $CT = [c_1 \cdots c_n]$, and

$$T^{-1}B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

and $R_i = c_i b_i$, one obtains the same results as found previously.

All these quantities, c_i, b_i, residues, and eigenvalues, may be obtained by inspection once we obtain the system model in modal coordinates. All effects of a higher-order mode discussed here may therefore be found numerically. Finally, we note that we obtain residues from manipulations involving the model matrix (eigenvectors), which thereby serve the same purpose as transfer-function numerators.

Analysis Approach

Although all the above ideas are available from linear systems theory, blind use of them will lead to incorrect answers to the fundamental questions cited at the outset. The straightforward modal analysis of this linearized math model of the aircraft dynamics alone ignores the salient characteristics of the actual disturbances exciting these dynamics in actual flight conditions. Two such disturbances of interest here are the pilot's control input and atmospheric turbulence. Neither of these may be treated as "white" random processes, for example. The significant characteristics of these "exciting subsystems" must be taken into account. In addition, important vehicle responses must be selected for meaningful analysis.

The most significant characteristic of the pilot, with regard to open-loop analysis of vehicle dynamics, is his finite bandwidth. To reflect this, one could consider his control inputs to the vehicle filtered through a low-pass filter, or

$$\frac{\delta_{\text{stick}}}{\delta_{\text{stick}_c}} = \frac{1}{\tau_p s + 1} \tag{4}$$

where the pilot's time constant τ_p may be taken to be the order of his neuromotor time constant, or 0.1-0.2 s. As a result, he will be incapable of significantly exciting modes above 20 rad/s.

The other input exciting the vehicle is atmospheric turbulence. If we use the Dryden gust model[4], for example, we could express it in the form

$$\dot{\bar{g}} = A_g \bar{g} + D_g n \tag{5}$$

where n is a white noise process of unit intensity.

Consider now the vehicle dynamics expressed as

$$\dot{\bar{x}}_v = A_v \bar{x}_v + B_v \bar{u}_v + D_v \bar{g}$$

where for longitudinal analysis one could choose $\bar{x}^T = [u, \alpha, \theta, \dot{\theta}, \eta_1, \ldots, \eta_n, \dot{\eta}_1, \ldots, \dot{\eta}_n]$ with $(u, \alpha, \theta, \dot{\theta})$ the standard rigid body degrees of freedom, or perturbed forward velocity, angle of attack, and rigid-body pitch attitude and rate. Likewise $\eta_1 \cdots \eta_n$ are the generalized coordinates of n in-vacuo structural vibration modes of the vehicle, with frequencies and mode shapes assumed available. However, to be discussed later, physical structural displacements and rates are preferred to generalized modal coordinates. The control surfaces define the control vector $\bar{u}_v$, while the effects of atmospheric gust turbulence determine the matrix D_v and the gust disturbance vector $\bar{g}$.

Collecting the two disturbance models and the vehicle model, one may write

$$\begin{bmatrix} \dot{\bar{u}}_v \\ \dot{\bar{g}} \\ \dot{\bar{x}}_v \end{bmatrix} = \begin{bmatrix} A_p & 0 & 0 \\ 0 & A_g & 0 \\ B_v & D_v & A_v \end{bmatrix} \begin{bmatrix} \bar{u}_v \\ \bar{g} \\ \bar{x}_v \end{bmatrix} + \begin{bmatrix} B_p \\ 0 \\ 0 \end{bmatrix} \bar{u}_c + \begin{bmatrix} 0 \\ D_g \\ 0 \end{bmatrix} n$$

or

$$\dot{\bar{x}} = \underline{A}\bar{x} + \underline{B}\bar{u}_c + \underline{D}n \tag{6}$$

where

A_p = pilot input dynamics, m controls, or diag$[-1/\tau_{p_1}, \ldots, -1/\tau_{p_m}]$, from Eq. (4)
B_p = diag$[1/\tau_{p_1}, \ldots, 1/\tau_{p_m}]$, from Eq. (4)
A_g, D_g = atmospheric turbulence model, from Eq. (5)

It is on this state model that the modal analysis must be performed, since it properly reflects the dynamic properties of the actual disturbance inputs.

Finally, the system output vector $\bar{y} = C\bar{x}$ must be selected to include vehicle responses important to the questions being addressed, or flexible vehicle handling characteristics. In the case of a longitudinal analysis, one could select the following responses to be evaluated.

$$\bar{y}^T = \left[\theta_R, \dot{\theta}_R, \theta_I, \dot{\theta}_I, \gamma, n_{z_p}\right]$$

where θ_R = rigid-body attitude, θ_I = indicated attitude at the

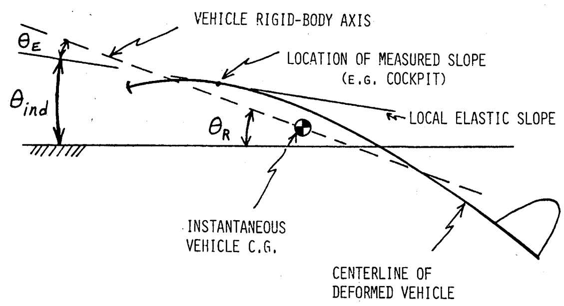

Fig. 1 Vehicle schematic.

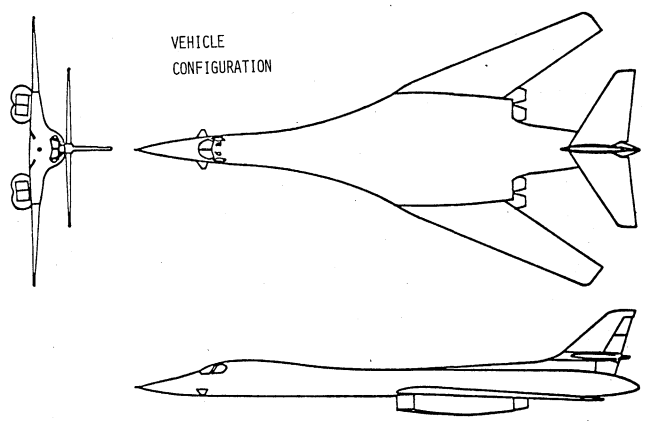

Fig. 2 Study vehicle geometry.

Table 1 Configuration summary

Configuration	In vacuo frequencies		Phugoid mode[a]	Short period mode[a]	Aeroelastic modes	
	Mode 1	Mode 2			First[a]	Second[a]
1(baseline)	13.7	21.2	(.02,.08)	(.53,2.8)	(.05,13.3)	(.02,21.4)
2	9.2	21.2	(0.,.06)	(.52,2.6)	(.09,8.8)	(.02,21.4)
3	6.2	21.2	(+.09)(−.08)	(.52,1.8)	(.20,5.9)	(.02,21.4)
4	13.7	4.8	(+.15)(−.13)	(.69,1.6)	(.05,13.3)	(.11,6.0)
5	10.7	9.3	(.05,.03)	(.55,2.4)	(.11,10.3)	(0.0,9.8)
6	11.7	11.7	(0.,.05)	(.54,2.6)	(.08,11.7)	(0.0,11.6)
7	6.9	6.9	(+.18)(−.15)	(.70,1.4)	(.19,7.3)	(0.0,6.9)

[a] Modal parameter notation, complex (ξ, ω_N), real $(-p)$, all frequencies in rad/s.

Table 2 Summary of experimental results

Configuration	RMS error, deg (mean $\pm 1\sigma$)	Cooper-Harper rating (mean $\pm 1\sigma$)	Comments
1	1.2 ± 0.6	1.6 ± 0.4	Very nice; no problem.
2	1.0 ± 0.5	2.0 ± 0.3	Little oscillation; more difficult than C1; slight control response lag.
3	5.7 ± 1.1	5.9 ± 1.9	Difficult; required high concentration; PIO problem; extreme response lag.
4	1.9 ± 0.3	3.1 ± 1.1	Little more difficult than C1; slightly sluggish attitude response.
5	1.2 ± 0.5	1.9 ± 0.4	Not difficult, little more oscillation, but could ignore it and fly rigid-body; like configuration 2.
6	1.5 ± 0.7	2.0 ± 0.5	Pretty good; same as 2.
7	7.6 ± 2.8	6.7 ± 1.6	With severe oscillations, virtually uncontrollable. Abrupt control inputs led to disaster.

cockpit, including elastic deformation, γ = flight path angle $(\theta_R - \alpha)$, and n_{z_p} = vertical acceleration at the cockpit. These responses are expressed in terms of the vibration mode shapes and generalized coordinates[5] as depicted in Fig. 1 or as

$$\theta_I = \theta_R - \sum_{i=1}^{n} \phi_\theta^i(l)\eta_i$$

$$n_{z_l} = U_0\dot{\gamma} + l\ddot{\theta}_R - \sum_{i=1}^{n} \phi_z^i(l)\ddot{\eta}_i$$

where $\phi_z^i(l)$ = magnitude of displacement mode shape at location l for mode i, and $\phi_\theta^i(l)$ = magnitude of mode slope at location l for mode i.

With these relations, one may express the output vector $\bar{y}$ in terms of the states of Eq. (6), or $\bar{y} = \underline{C}\bar{x}$. The analysis follows.

Experimental Data

The data to be considered were obtained via fixed-base simulation of a pursuit attitude tracking task, using a family of vehicle dynamic models.[6] The baseline model was representative of a vehicle as shown in Fig. 2, and is documented in Ref. 7. The other models are parametric variations of the baseline, obtained by reduction of the in vacuo vibration frequency of the first and/or second fuselage bending mode, denoted mode 1 and mode 2, respectively. The mode shapes were assumed constant and aeroelastic vehicle models derived for each case. (One might consider all these configurations then as having the geometry as in Fig. 2, but the material properties of the vehicle are changed in such a way so as to result in the different structural vibration frequencies.) A summary of seven of the configurations, listed in terms of the vibration modal frequencies assumed and the resulting longitudinal eigenvalues is given in Table 1. Note that the identification of the aeroelastic modes as "mode 1" and "mode 2" relates these two vehicle modes to their corresponding in vacuo vibration mode.

Experimental tracking data were obtained for all configurations with four subjects (pilots). The display included a fixed reference, a command bar, and an attitude bar, all displayed on a CRT. The vertical position of the command bar indicated "commanded" pitch attitude, θ_c, while the vertical position of the attitude bar (above or below the reference) indicated the vehicle attitude, including the elastic slope at the cockpit relative to the rigid-body vehicle axis, or θ_I, defined in the previous section. It is important to note that the rigid-body attitude θ_R may be thought of as the vehicle attitude, and this is the response the pilot is attempting to control. (See pilot comments in Table 2.) However, θ_R must be estimated from θ_I, the displayed variable. For further discussion, Ref. 8 includes a closed-loop, pilot/vehicle analysis of this data.

Listed in Table 2 are subjects' comments corresponding to these seven specific configurations, along with the statistics on tracking error and Cooper-Harper[9] subjective ratings. Note that the subjects' comments indicate that the presence of the oscillations were perceived in all cases, except configuration one, and that in the case of configurations 3 and 7, the dynamics were completely unacceptable.

Analysis of the Data

Returning to the fundamental question posed in the Introduction, it should be clear that aeroelastic effects did affect the handling characteristics, significantly, as evaluated in the simulation of the pitch-tracking task. So significant were these effects, in fact, that in the cases of configurations 3 and 7, the dynamics received Level 3 ratings, while the baseline configuration was Level 1. The question now to be addressed is, "How does this come about?"

First, note that "static" aeroelastic effects alone are not the problem here since the results are not explained from the characteristics of the short-period and phugoid modes. Specifically, note that Configuration 4 received a considerably better subjective rating than Configuration 3, which was rated almost Level 3. Yet Configuration 4 had only a slightly lower short-period frequency and an even more unstable phugoid mode than did Configuration 3. Reduced-order, "static-elastic" models were also obtained for these two configurations by modal residualization (sometimes also referred to as the singular perturbation method[10]). The eigenvalues for the reduced-order Configuration 3 are 0.07, -0.05, and $-1.02 \pm 1.48\,j$ rad/s. Those for the reduced-order Configuration 4 are 0.16, -0.13, and $-1.17 \pm 0.81\,j$ rad/s. For a T_{θ_2} of 1.25 s, Ref. 11 indicates Level 1 attitude dynamics for both cases.

Furthermore, the eigenvalues associated with the two dynamic aeroelastic modes would not appear to explain the handling characteristics, since the lowest aeroelastic mode frequencies and dampings for these same two configurations are comparable.

A much clearer picture is obtained from a modal analysis, as described in the previous sections. We will evaluate each of the vehicle responses in the chosen "output" vector $y^T = [\theta_R, \dot\theta_R, \theta_I, \dot\theta_I, \gamma, n_{z_p}]$, where n_{z_p} is the vertical acceleration at the cockpit location. It is noted that although the methodology will be used to "explain" fixed-base simulation results in this situation, maximum utility of such analysis might be realized by applying it to select the appropriate dynamic model prior to a simulation. By so doing, the significant modes contributing to the acceleration response, for example may be determined. Care should be taken to include them in a motion simulation.

We determine the elements of the modal controllability matrix $T^{-1}\underline{B}$ and disturbability matrix $T^{-1}\underline{D}$, where $\underline{B}$ and $\underline{D}$ are as defined in Eq. (6). Also, we obtain the modal observability matrix $\underline{C}T$, corresponding to the output vector y selected. From these the residues corresponding to an impulsive pilot elevator command δ_{E_c} (through A_P and B_P), as well as the residues corresponding to an impulsive input to the gust turbulence model, or n, then may be obtained. (Note in the following discussion, we will refer to these as the "pilot impulse response" and the "gust impulse response," respectively.) Now the contribution of each mode to these meaningful system time responses may be assessed.

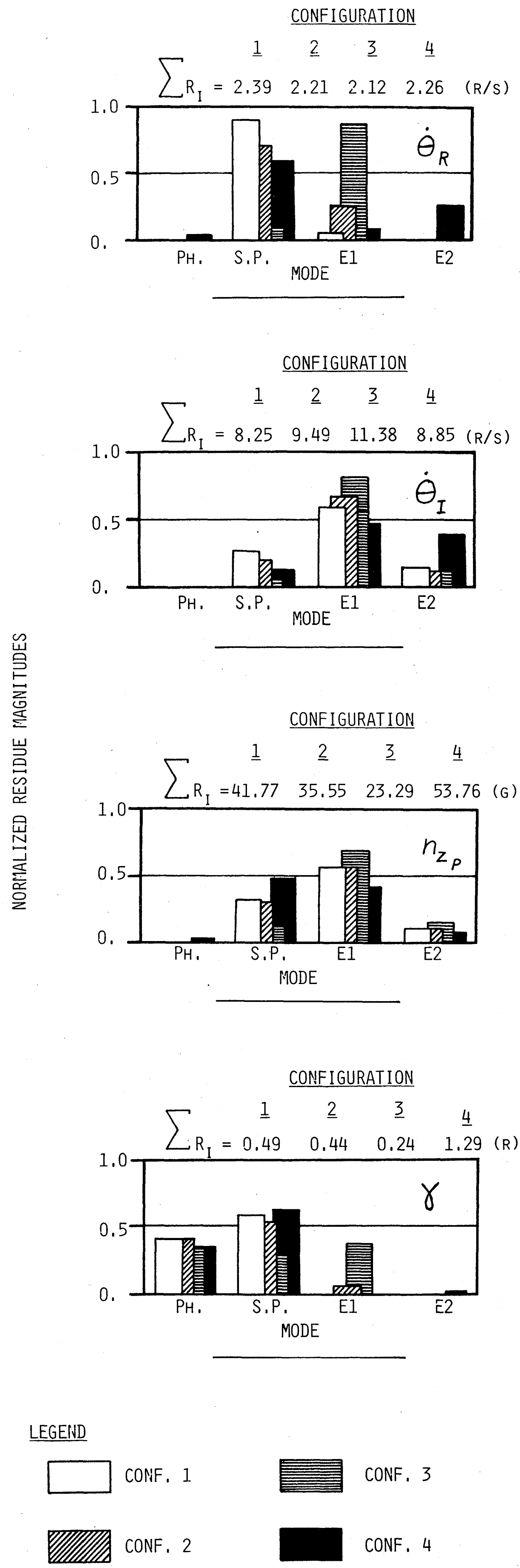

Fig. 3 Pilot impulse residues, configurations 1-4.

The residues for the pilot impulse response for four of the seven cases of vehicle dynamics (Configurations 1-4) are shown in Fig. 3. Plotted are the normalized relative magnitudes of the modal residues for each vehicle mode, which has been identified from consideration of its eigenvector, or "mode shape." The residue magnitudes have been normalized such that they sum to unity for comparison purposes. Also, the summation of all vehicle modal residue magnitudes for the response is given with each figure. So the absolute magnitude for each mode may be calculated if desired, since the normalized value reflects the fraction of the total response associated with the mode.

Initial attention will be focused on the residues in the rigid and total-indicated attitude responses as these are of primary importance to our pitch tracking experimental results. The greater contribution of the aeroelastic dynamic modes (E1 and E2) in $\dot{\theta}_I$ than in $\dot{\theta}_R$ is clearly evident, and is as expected since $\dot{\theta}_R$ is the response of the rigid-body motion of the vehicle (see Fig. 1). However, for Configurations 1-3, the monotonic increase in the first flexible mode's contribution to the rigid attitude ($\dot{\theta}_R$) response should be noted, as the natural vibration frequency of the first elastic mode is reduced (see Table 1 for reference). Perhaps the most significant result of this analysis is the revelation that in Configuration 3, the rigid-body attitude response to pilot input is essentially dominated by the first ($\zeta = 0.2$, $\omega_n = 5.9$ rad/s) aeroelastic mode. It is clear from this result why the pitch tracking results were poor, and it is also clear that these dynamics could not be well modeled with a reduced-order rigid-body model, as it would not include this mode's dynamic response.

In contrast to the above situation, consider the result for Configuration 4. Although the contribution of the lower frequency aeroelastic mode (or E2 in this configuration, see Table 1) is significant, it does not dominate the rigid-body attitude response of the vehicle, as in Configuration 3. It is this fact that would explain the better tracking performance and rating for this configuration compared to Configuration 3.

Finally, note that in all the remaining configurations, the tracking performance and ratings agreed with the trends in the elastic modes' contributions, as indicated by the residues, to the vehicle's rigid attitude response.

Turning the attention briefly to the flight path γ and cockpit acceleration n_{z_p} results, the significance of the aeroelastic modes in the acceleration is clearly seen in all cases. Of course this response is not perceivable in a fixed-base simulation, as noted previously, but these results should be taken into account in planning moving-base simulations of these types of vehicles. And finally, we see in Configuration 3 that the aeroelastic mode also contributed significantly to the flight-path response of the vehicle to pilot impulse. Clearly, any attempt to describe the motion of this configuration only in terms of its rigid-body modes would be incorrect.

Table 3 Modal analysis results—Configuration 1

Mode	Eigenvalue	Controllability[a] by δ_{P_C} 1/s	Disturbability[a] by n_{GUST},[b] $\sqrt{1/s}$	Output observability[a] $\dot{\theta}_R$, r/s	$\dot{\theta}_I$, r/s	n_{Z_P}, g's
Phugoid	$-5.8E\text{-}04 \pm j8.7E\text{-}02$	0.11	0	0.080	0.08	2.34
Short-period	$-1.3 \pm j2.4$	1.24	0.0015	0.880	0.91	5.63
Elastic 1	$-.66 \pm j13.0$	2.11	0.0038	0.042	1.13	5.26
Elastic 2	$-.47 \pm j21.$	0.53	0.0057	0.018	1.05	4.26

[a]Magnitude only, [b]$\sigma_{w_G} = 1$ ft/s in turbulence model.

Table 4 Modal analysis results—Configuration 2

Mode	Eigenvalue	Controllability[a] by δ_{P_C}, 1/s	Disturbability[a] by n_{GUST},[b] $\sqrt{1/s}$	Output observability[a] $\dot{\theta}_R$, r/s	$\dot{\theta}_I$, r/s	n_{Z_P}, g's
Phugoid	$1.1E\text{-}03 \pm j7.7E\text{-}02$	0.10	0	0.070	0.07	2.03
Short period	$-1.2 \pm j2.2$	0.96	0.0013	0.820	0.76	6.03
Elastic 1	$-.72 \pm j8.8$	2.96	0.0044	0.098	1.16	3.15
Elastic 2	$-.46 \pm j21.$	0.50	0.0057	0.018	1.04	4.15

[a]Magnitude only, [b]$\sigma_{w_G} = 1$ ft/s in turbulence model.

Table 5 Modal analysis results—Configuration 3

Mode	Eigenvalue	Controllability[a] by δ_{P_C}, 1/s	Disturbability[a] by n_{GUST},[b] $\sqrt{1/s}$	Output observability[a] $\dot{\theta}_R$, r/s	$\dot{\theta}_I$, r/s	$n_{\dot{Z}_P}$, g's
Phugoid	-0.052&0.072	0.038&0.053	$1.5E\text{-}05$&$2.5E\text{-}05$	0.037&0.060	0.05&0.046	1.75&1.08
Short period	$-0.78 \pm j1.5$	0.20	0.0007	0.605	0.45	7.50
Elastic 1	$-1.05 \pm j5.86$	3.90	0.0050	0.230	1.28	1.94
Elastic 2	$-0.46 \pm j21.3$	0.49	0.0056	0.018	1.03	4.11

[a]Magnitude only, [b]$\sigma_{w_G} = 1$ ft/s in turbulence model.

Table 6 Modal analysis results—Configuration 4

Mode	Eigenvalue	Controllability[a] by δ_{P_C}, 1/s	Disturbability[a] by n_{GUST},[b] $\sqrt{1/s}$	Output observability[a] $\dot{\theta}_R$, r/s	$\dot{\theta}_I$, r/s	n_{Z_P}, g's
Phugoid	0.16& -0.13	0.34&0.25	$3.05E\text{-}05$&$3.19E\text{-}05$	0.14&0.11	0.18&0.072	0.297&4.43
Short period	$-0.84 \pm j1.04$	1.45	0.0005	0.472	0.88	9.3
Elastic 1	$-0.70 \pm j13.3$	2.03	0.0030	0.046	1.01	4.9
Elastic 2	$-0.56 \pm j5.93$	1.45	0.0060	0.212	0.68	1.4

[a]Magnitude only, [b]$\sigma_{w_G} = 1$ ft/s in turbulence model.

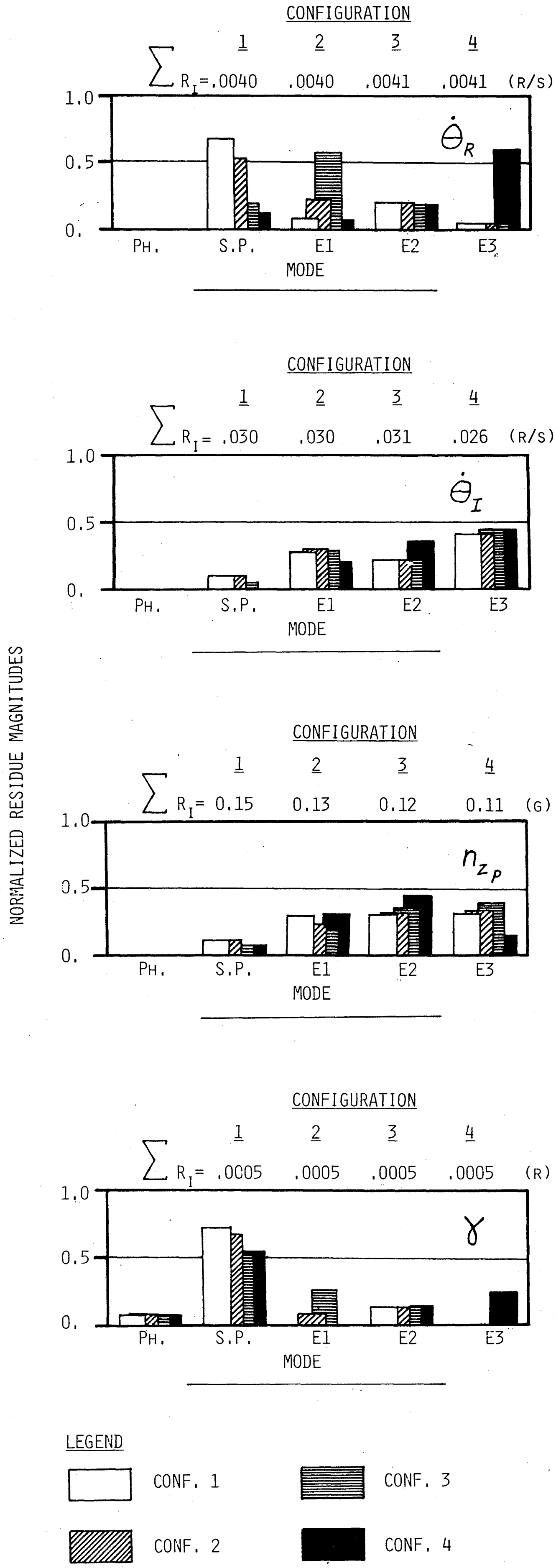

Fig. 4 Gust impulse residues, configurations 1-4.

An evaluation of the modal controllability (pilot input), disturbability (gust input), and observability in the vehicle responses reveals more basically the same results presented in terms of residues. Given in Tables 3-6 are these results for the first four vehicle configurations. It is important to note that unlike the residues, the values listed in these tables depend on the particular normalization of the eigenvectors and state definition. These values in Tables 3-6 were obtained after normalizing all eigenvectors to unity length. In addition, it is important to use physical states in the state equations, and to select units of approximately equal significance. Therefore, elastic mode slope at the cockpit rather than a mode's generalized coordinate was selected as the modal state. The data in these tables show, for example, that although the first elastic mode in Configuration 1 may be easily excited by pilot inputs, the "mode shape" or eigenvector is such that this mode exhibits relatively little rigid-body attitude motion, as indicated by the fact that it is relatively unobservable in rigid-body pitch rate. This is not the case, however, in cockpit acceleration. This mode is highly observable in that vehicle response variable. Again note the changes in these characteristics as the free vibration frequencies are varied among these configurations, with special attention to Configurations 3 and 4.

To complete this analysis, the residues for the gust impulse response for these configurations are presented in Fig. 4, in the same format as the pilot impulse response. The vehicle model in this case, however, was expanded to include the first wing bending vibration mode (denoted E2), since this mode could be significant in vehicle response to turbulence. The eigenvalue of the ccorresponding aeroelastic mode remained approximately constant at $-4.5 \pm 22.j$.

In these results, we observe a similar trend of increased contribution from the aeroelastic modes as the vibration frequencies of modes E1 and/or E3 are varied among configurations. It is significant to note that again, for Configuration 3, the elastic mode contribution to rigid-body attitude is most significant, thus indicating that attitude control in turbulence would also be a difficult task for this vehicle.

A similar situation is revealed in the results for Configuration 4. Although pitch tracking performance for this configuration was not as bad as that for Configuration 3, these results show that its (Configuration 4's) turbulence response would be more like that of Configuration 3 since the rigid-body attitude response is also dominated by the low-frequency aeroelastic modes.

Summary and Conclusions

A multivariable modal-analysis approach has been presented for evaluating the aircraft dynamics, focusing on the meaningful vehicle response to pilot inputs and atmospheric turbulence. From application of this method it is hoped that the mechanisms governing how dynamic aeroelastic modes affect the vehicle response, with special attention to the handling qualities, may be better understood. Although modal analysis is the tool, vehicle time response is emphasized.

In evaluating previously obtained experimental data for a family of vehicle dynamic models, it is shown that flexible aeroelastic effects did significantly affect the handling qualities in the simulated pitch tracking task, and that consideration of only the eigenvalues of both rigid and aeroelastic modes could not explain the piloted vehicle results. More thorough analysis revealed, however, that although the lowest aeroelastic mode frequency was still three times greater than the short-period frequency, the all-important rigid-body attitude response was dominated by the aeroelastic mode, with dominance defined in terms of relative magnitude of the modal residues. This (residue) information is readily obtained from numerical analysis of the linear, time-domain system model, and tends to tie together the significance of eigenvectors in the time domain and transfer function numerators in the frequency domain.

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